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Convection in a porous thermosyphon imbedded in a conducting medium

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Abstract—In a closed geological thermosyphon, thermal convection in a closed loop is coupled to conduction in the surrounding earth. Heat flow from an isolated source in such a porous thermosyphon is studied using approximate analysis and numerical simulation. Particular problems associated with using numerical elements that are of the same size as the structural detail are identified. A Rayleigh number definition identical with Bejan's is obtained, except for an additional ratio of a/c where a is a characteristic dimension describing the minor cross-section of the thermosyphon loop and c a characteristic dimension for the major diameter of the loop. At a Rayleigh number above 1, convection results in a temperature reduction near the source.

1. INTRODUCTION

A THERMOSYPHON is a loop, either open or closed, in which fluid circulates due to natural convection caused by a heat source in the lower part of the loop. Thermosyphons are of interest because they have been successfully applied to a variety of technological problems [1]. Most of these applications are above the ground, such as in solar hot water systems, the cooling of transformers and car engines. Below the ground surface, it has been surmised that some geophysical processes may contain natural thermosyphons. For example, Torrance [2] has studied open-loop thermosyphons in the context of hydrothermal circulation in the oceanic and continental crusts. Thermosyphons may play a role in the formation of ore bodies, and also may occur either intentionally or unwittingly in the underground storage of nuclear waste [3].

Studies of thermal convection in fluid saturated porous media have often assumed that the properties of the porous media are uniform. Only a few examples of heterogeneous systems have been investigated, such as the studies by McKibbin and O'Sullivan [4, 5] of a layered medium. Heterogeneity abounds in nature, and it is possible to create theoretically or describe through observation many different heterogeneous formations. Obviously one study cannot consider all types of heterogeneity, thus the thermosyphon provides a convenient single category of heterogeneity for study. A thermosyphon can exist when a loop of high permeability rock occurs within rock of lower permeability. We have limited our study to the case where the permeability of the surrounding medium is much less (more than four orders of magnitude) than the permeability within the thermosyphon. Thus convection in the thermosyphon is significant long before convection in the surrounding medium becomes apparent.

Convection from a point source into an infinite medium is simple because there is no length scale to characterize the problem other than the location of the point of observation relative to the source point [6–8]. Convection from a spherical cavity is also simple because the problem can be described with only one length scale (the radius). However, the thermosyphon is complicated because it has many associated length scales: the height of the loop; the width of the loop; the cross-sectional area of the loop; and the length scale(s) associated with the external boundaries of the problem.

Normally, the wall of a thermosyphon is assumed to be insulating, except over short lengths where heat enters or is extracted. In the geologic thermosyphon, the wall is the boundary of a continuous heat conducting (and possibly convecting) region through which heat escapes. This complicates the formulation of the problem because the heat conduction in the surrounding medium is intimately coupled with the convection in the loop.

To study the geologic thermosyphon we have performed a number of numerical experiments. These simulations determine the behaviour of a porous thermosyphon as a function of the various length scales involved. An integral part of the problem as we examine it here is that the size of the elements is important: normally one would choose to use much smaller elements but this can be computationally impractical. The significance of the choice of element size is discussed as the analysis proceeds. We have quantified the performance of the thermosyphon in terms of the average temperature in the element containing the heat souce instead of using Nusselt numbers or other measures of heat transfer. This was because of our particular interest in isolated heat sources, possible chemical reaction rates and maximum thermal stresses.

The study of thermosyphons has become fashionable over the last few years due to the observation

NOMENCLATURE

a	radius of spherical element, or the radius of a sphere inscribed in a cubic element [m]	$\langle T \rangle$	temperature within the element containing a heat source [K]
b	radius of spherical boundary, or the radius of a sphere inscribed in a cubic boundary [m]	v	Darcy velocity.
c	thermosyphon side length [m]		Greek symbols
C	specific heat [J kg^{-1} K^{-1}]	α	thermal diffusivity, $\kappa/\rho C$ [m^2 s^{-1}]
d	distance from point source [m]	β	coefficient of thermal expansion [K^{-1}]
$\mathbf{g}$	gravitational acceleration vector [m s^{-2}]	κ	thermal conductivity [W m^{-1} K^{-1}]
k	permeability [m^2]	μ	dynamic viscosity [N s m^{-2}]
p	pressure [N m^{-2}]	ν	kinematic viscosity [m^2 s^{-1}]
Q	strength of heat source [W]	ρ	density [kg m^{-3}]
r	distance from point source [m]	τ	temperature difference caused by convection [K]
Ra	Rayleigh number	ϕ	porosity.
t	time [s]		Subscript
T	temperature above reference or background [K]	0	reference value.

that oscillatory or chaotic flow may occur under appropriate conditions [9–13]. These conditions involve heat fluxes well above values to be expected in geophysical applications, thus we have not considered them.

2. THEORETICAL BACKGROUND

It is instructive to commence by considering heat flow from isolated sources in homogeneous media. This provides a framework to introduce sequentially the various parameters that we used for describing heat flow in a thermosyphon.

For conduction in a homogeneous body, it is only necessary to solve the equation

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \tag{1}$$

where T is the temperature and α the thermal diffusivity of the body. Convection requires the incorporation of fluid motion and some additional equations. The equations for continuity, motion and thermal transport for convection in a fluid saturated porous medium are, respectively

$$\nabla \cdot \rho v = -\frac{\partial \phi \rho}{\partial t} \tag{2}$$

$$\frac{\mu}{k} v = -\nabla p - \rho \mathbf{g} \tag{3}$$

$$\frac{\partial T}{\partial t} + v \cdot \nabla T = \alpha \nabla^2 T \tag{4}$$

where v is the Darcy velocity of the fluid, k the permeability of the medium, μ the dynamic viscosity of the fluid, p the pressure, $\mathbf{g}$ the gravitational acceleration vector, $\rho = \rho(T)$ the density of the fluid, and α is now the thermal diffusivity of the saturated porous medium. For slightly compressible liquids it is convenient to use the relationship

$$\rho = \rho_0(1-\beta T) \tag{5}$$

for the density, where β is the coefficient of thermal expansion and ρ_0 a reference density when $T = 0$.

2.1. *Steady conduction from a spherical source*

The easiest relevant problem to begin with is steady conduction from a point source of strength Q into an infinite medium of thermal conductivity κ. The solution gives the temperature T at a distance r from the source as

$$T = \frac{Q}{4\pi\kappa r} \tag{6}$$

where $T \to 0$ as $r \to \infty$.

In a numerical method such as finite elements or integrated finite differences, temperatures averaged over a region are calculated rather than temperatures at single points. Neglecting for the moment that numerical methods use polyhedra (such as cubes) to fill space instead of spheres, consider a spherical region of radius a centred on a point source. The average temperature $\langle T \rangle$ within this region is determined from

$$\langle T \rangle = \frac{3}{4\pi a^3} \int_0^{2\pi} \int_0^{\pi} \int_0^{a} T(r, \theta, \phi) r^2 \sin\theta \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}\phi \tag{7}$$

in spherical coordinates. Substituting equation (6) into equation (7) gives

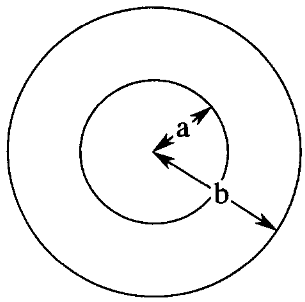

FIG. 1. Definitions of the length scales involved in defining concentric spheres.

$$\langle T \rangle = \frac{3Q}{8\pi\kappa a} \quad (8)$$

as the average temperature within the sphere. If the sphere for averaging had not been centred on the source, but had been centred a distance d away from the source, then the average temperature within the sphere could be determined from

$$\langle T \rangle = \frac{3}{4\pi a^3}\int_0^{2\pi}\int_0^{\pi}\int_0^{a}\frac{Q}{4\pi\kappa} \times \frac{r^2 \sin\theta}{\sqrt{(r^2+d^2-2rd\cos\theta)}}\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi \quad (9)$$

which gives the result

$$\langle T \rangle = \frac{Q}{8\pi\kappa a^3}(3a^2-d^2) \quad \text{if} \quad d<a \quad (10)$$

and the result

$$\langle T \rangle = \frac{Q}{4\pi\kappa d} \quad \text{if} \quad d>a. \quad (11)$$

Thus the average temperature falls from a maximum when the averaging sphere is centred on the source, first in a parabolic fashion while the source is within the averaging sphere ($d < a$), then as the inverse first power of the distance when the source is outside the averaging sphere ($d > a$).

2.2. *Conduction to a finite spherical boundary*

Infinite media are never real, and infinite meshes are difficult to create for numerical simulation. Thus it is worthwhile to consider conduction to a finite spherical boundary of radius b fixed at temperature $T = 0$ (Fig. 1). The average temperature within the inner sphere of radius a is given by

$$\langle T \rangle = \frac{Q}{4\pi\kappa}\left[\frac{3}{2a}-\frac{1}{b}\right]. \quad (12)$$

2.3. *Convection from a point source*

When studying thermal convection in porous media, the usual approach to obtain analytical solutions has been to use the Boussinesq approximation in which density changes are accounted for only in the buoyancy term in the equation of motion (equation (5) is used in equation (3), but not in equation (2)). There have been few studies of convection from isolated sources or in unbounded media. An exception to this has been Wooding [14], who considered a horizontal line source of thermal energy located in an infinite, fluid-saturated porous medium.

More recently, Bejan [6] determined the small Rayleigh number approximation for convection around a point source in an infinite medium. He defined a Rayleigh number as

$$Ra = \frac{Qgk\beta}{\alpha^2\mu C} \quad (13)$$

where Q is the energy generated per unit time, g the gravitational acceleration, k the permeability, β the coefficient of thermal expansion, α the effective thermal diffusivity, μ the dynamic viscosity and C the specific heat of the fluid. Bejan's solution for the temperature in spherical coordinates is

$$T = \frac{Q}{4\pi\kappa r}\left[1+\frac{1}{8\pi}\cos\theta\, Ra + 5\cos 2\theta\, Ra^2 + \frac{1}{55\,296\pi^2}\cos\theta(47\cos^2\theta-30)Ra^3+\cdots\right] \quad (14)$$

where T is measured above the temperature at $r \to \infty$. In the absence of any boundary surface that can inhibit motion, any deviation from an isothermal state will result in fluid motion. There is not a critical Rayleigh number below which convection does not occur, only a value at which it becomes insignificant.

As we have indicated, when using numerical methods based on discretizing space such as finite differences or finite elements, the temperature at an arbitrary infinitesimal point cannot be determined. Rather, the averages within many regions (the elements) are calculated. In this vein, it was of use to determine the average temperature within a sphere of radius a centred on the point source. When equation (14) is substituted into equation (7) the following equation can be obtained from Bejan's solution:

$$\langle T \rangle = \frac{3Q}{8\pi\kappa a}\left[1-\frac{5}{2304}\frac{Ra^2}{\pi^2}+O(Ra^4)\cdots\right]. \quad (15)$$

A numerical method with an element centred on the point source should give an answer similar to equation (15) for the average temperature in the element. Thus a numerical method that discretizes at a different length scale a will give a different value of $\langle T \rangle$, but the value of Ra will not be different. Note that convection starts to become important when $Ra > 10$.

Hickox and Watts [7] have studied steady thermal convection from an isolated source in a porous medium by using similarity transforms. They obtained a number of numerical results from which velocity and temperature fields can be constructed for a specific set of Rayleigh numbers.

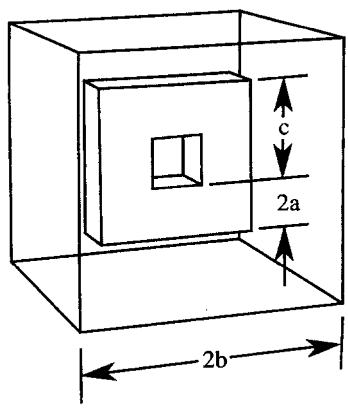

FIG. 2. Definitions of the length scales involved in defining a thermosyphon imbedded in a cube.

2.4. *Convection between two surfaces*

Convection between two horizontal surfaces is characterized by the distance between the two surfaces L. This length scale appears in the definition of the Rayleigh number for such a configuration when heat is emitted from the bottom surface [15]

$$Ra = \frac{\Delta T g k \beta L}{\alpha \nu} \tag{16}$$

where ΔT is the temperature difference between the plates and ν the kinematic viscosity. A similar definition occurs for convection bounded by vertical coaxial cylinders, where L is identified as the distance between the cylinders [16, 17]. For the thermosyphon it would be expected that the additional length scales would comprise part of the Rayleigh number definition. The question is to resolve exactly how they appear in the definition.

3. CONVECTION IN A THERMOSYPHON

To describe a thermosyphon it is first necessary to define some length scales. This can be done by extending the existing definitions for a sphere and a cube (Fig. 2). It is convenient for the numerical simulations to have the quantities b and c an integral number of multiples of a.

In the geologic thermosyphon, the formulation of the problem is complicated because the heat transport in the surrounding medium is intimately coupled with transport in the thermosphon loop. Like Torrance [2], we used the simplification that heat in the surrounding rock was assumed to travel only by conduction. However, Torrance introduced a further simplification in which the surrounding medium had a high thermal conductivity so that the thermosyphon wall had a temperature approaching the ambient geothermal gradient. We did not make such an assumption.

If steady-state conditions exist, the source strength Q will equal the heat flow out of the element containing the source. Heat transport out of the element containing the source occurs through two mechanisms, conduction and convection

$$Q = Q_{\text{conduction}} + Q_{\text{convection}}. \tag{17}$$

Conduction has already been discussed, and is quantified by equation (8). When fluid circulates, warm fluid will leave the source element and be replaced by colder fluid. This heat transport due to convection, $Q_{\text{convection}}$, will be the difference in the rate of heat leaving the element and the heat entering the element. When the fluid motion is relatively small, this can be determined from the temperatures in the elements adjacent to the element containing the source given by equation (11) with $d = 2a$, namely $\langle T \rangle/3$. Thus, providing that convection is small, the heat flow out of the element containing the source will be approximately

$$Q = \frac{8\pi\kappa a}{3}\langle T \rangle + \tfrac{8}{3} v \rho C a^2 \langle T \rangle \tag{18}$$

where v is the Darcy velocitiy of fluid flow around the loop (the loop has cross-sectional area $4a^2$).

The steady velocity of the fluid around the thermosyphon loop occurs when the viscous drag balances the buoyancy driving force. In a continuum this could be written as

$$v = \frac{k}{4\mu c}\oint \rho_0 \beta T(l) \mathbf{g}\, \mathrm{d}l \tag{19}$$

where the integral dl is taken around the closed thermosyphon loop and v is averaged across the cross-section of the thermosyphon loop. The Boussinesq approximation means that v is constant around the loop. In a square thermosyphon the two horizontal segments will not contribute to the buoyancy, so only the vertical segments need to be included

$$v = \frac{k}{4\mu c}\left[\int_0^c \rho_0 \beta T(l) g\, \mathrm{d}l - \int_{2c}^{3c} \rho_0 \beta T(l) g\, \mathrm{d}l\right]. \tag{20}$$

A further assumption is to assume that most of the additional heat has been lost when the fluid gets half-way around the loop, thus only the first term in equation (20) is important. If heat transport is dominated by conduction, the average temperatures in the elements above and including the heat source can be summed. For example

$$v = \frac{ka}{2\mu c}\rho_0 \beta g\left(\langle T \rangle + \sum_{n=1}^{2na \leqslant c} \frac{\langle T \rangle}{3n}\right). \tag{21}$$

Thus, for three elements in the vertical arm containing the source

$$v = \frac{3ak\rho_0 \beta g \langle T \rangle}{4\mu c}. \tag{22}$$

The numerical factor would vary only slightly if we had summed over four elements rather than three (7% variation). Substituting equation (22) into equation (18) gives

$$\frac{2\rho_0^2 C\beta gka^3}{\mu c}\langle T\rangle^2+\frac{8\pi\kappa a}{3}\langle T\rangle-Q=0. \quad (23)$$

Providing that the coefficient of $\langle T\rangle^2$ is small, a series expansion of the positive root can be obtained

$$\langle T\rangle=\frac{3Q}{8\pi\kappa a}-\frac{27\rho_0^2C\beta gkQ^2}{256\mu c\pi^3\kappa^3}-O\left[\frac{\rho_0^2C\beta gkQ^{3/2}}{\mu\kappa^{5/2}}\right]^2. \quad (24)$$

Thus, convection becomes more important than conduction when

$$\frac{9\rho_0^2C\beta gkQa}{32\mu c\pi^2\kappa^2}>1. \quad (25)$$

The variables in this ratio define a Rayleigh number for the thermosyphon as

$$Ra=\frac{Q\beta gka}{\alpha^2\mu Cc}. \quad (26)$$

This is the same as equation (13) except for the additional ratio a/c. Using this definition of Rayleigh number equation (24) can be written as

$$\langle T\rangle=\frac{3Q}{8\pi\kappa a}\left[1-\frac{9}{32\pi^2}Ra-O[Ra]^2\right]. \quad (27)$$

This equation gives an estimate of the performance of the thermosyphon for small Rayleigh number. Larger Rayleigh numbers are considered below.

4. NUMERICAL STUDIES

Beyond the simple results above, progress has only been made with sophisticated numerical techniques.

A number of numerical codes have been written to simulate fluid and heat flow in rock. Several of these have been described in a review by O'Sullivan [18]. SHAFT, MULKOM and TOUGH originated at the Lawrence Berkeley Laboratory [19]. We chose to use SHAFT79 for the present work because it has received widespread application and testing, and it is well documented [20, 21]. Based on the integrated finite-difference method, SHAFT79 (Simultaneous Heat and Fluid Transport) solves the coupled equations (2)–(4) for heat and fluid flow in porous rock.

4.1. *Simulation specifics*

All the simulations reported here were run with uniform initial temperature conditions of 323 K (50°C). During the simulations the outer boundary was maintained at 323 K, and all temperatures identified by T in this section are relative to this reference value. The following constants were assigned to the rock:

density	$\rho_0 = 2600$ kg m^{-3}
porosity	$\phi = 0.10$
thermal conductivity	$\kappa = 2.1$ W m^{-1} K^{-1}
specific heat	$C = 1000$ J kg^{-1} K^{-1}
thermal diffusivity	$\alpha = 8.1\times10^{-7}$ m^2 s^{-1}

where $\alpha = \kappa/\rho C$.

SHAFT79 uses a table for fluid properties that are a function of temperature. The coefficient of thermal expansion β is approximately 5×10^{-4} K^{-1} for water around 323–333 K, and the viscosity of water μ is approximately 1.0 mN s m^{-2}.

We found that for a design feature as complex as the thermosyphon, which has only one plane of symmetry, at least 196 elements were required for meaningful results in three dimensions. This number is determined in the following manner. In one dimension, for n elements that have a temperature that can vary, $n+2$ elements are required, which includes the fixed temperature elements at the two ends. In three dimensions, this translates to $(n+2)^3$ elements. But the thermosyphon does have one plane of symmetry which can be used to reduce the number of elements to $(n+2)^2(n+3)/2$. At least one element is required for the centre of the thermosyphon, a layer for each side of the loop, and a layer outside the loop. Thus the minimum value for $n = 5$, corresponding to $7\times7\times4 = 196$ elements. If $n = 7$, 405 elements are required.

Simulations were run until both the maximum internal energy change and the maximum density change were below predetermined values. Although not a true steady state, this represents an effective steady state. The default value for energy change was set at 1.0 J m^{-3} and the default value for density change was 1×10^{-4} kg m^{-3}.

4.2. *Steady conduction from a cubic source*

As we have already indicated, numerical methods use polyhedra to fill space rather than spheres. Thus, the simplest relevant problem for numerical study is conduction from a hollow cube. With a cubic outer boundary this problem has three planes of symmetry, thus only one-eighth of the region of interest needs to be simulated. We may reasonably guess that the hollow cube with outer side length $2b$ and inner side length $2a$ would be similar to the hollow sphere solution above, except the coefficients of a and b. That is, we expect an equation of the form

$$\langle T\rangle=\frac{Q}{4\pi\kappa}\left[\frac{3\gamma_1}{2a}-\frac{\gamma_2}{b}\right] \quad (28)$$

with the constants γ_1 and γ_2 to be determined. We performed a series of simulations with b held constant to determine the coefficient of a, then we performed a series of simulations with a held constant to determine the coefficient of b. In this way, from numerical experiments with SHAFT79, we obtained the result

$$\langle T\rangle=\frac{Q}{4\pi\kappa}\left[\frac{1.60}{a}-\frac{1.05}{b}\right]. \quad (29)$$

This means that the larger the inner cavity, the cooler the temperature within the cavity, assuming that the

Table 1. The formula $\langle T\rangle = (Q/4\pi\kappa)[(1.60/a)-(1.05/b)]$ compared to numerical simulations for convection in a cube with $c = 6a = 6b/5$. The accuracy of the formula is shown in the table

$c = 6a = 6b/5$	$(Q/4\pi\kappa)[(1.60/a)-(1.05/b)]$	$\langle T\rangle$ Simulated	Difference
18.0	35.2	35.2	0.0
27.0	23.5	23.5	0.0
36.0	17.6	17.6	0.0
45.0	14.1	14.0	0.1
54.0	11.7	11.7	0.0
72.0	8.80	8.81	0.01

temperature within the cavity is uniform. With common numerical methods, such as the one we have used, distances smaller than the element size are not resolved (as we have already discussed). We may assume that the inner cavity is either highly conducting, or contains well-mixed fluid. In these numerical simulations we used a heat source $Q = 8$ kW. With a thermal conductivity $\kappa = 2.1$ W m^{-1} K^{-1}, this gives $Q/4\pi\kappa = 304$ m K. The results shown in Table 1 indicate the accuracy of equation (29) for determining the temperature in the source element.

This solution for conduction from a cubic source can now be used as a reference for convection problems, as all convection problems we considered have a conduction component based on the first term in equation (27). Temperatures relative to the equivalent conduction problem are denoted by τ, where τ is defined by

$$\tau = \frac{Q}{4\pi\kappa}\left[\frac{1.60}{a} - \frac{1.05}{b}\right] - \langle T\rangle. \tag{30}$$

Equation (30) is of the same form as equation (15).

5. STEADY CONVECTION IN A THERMOSYPHON

Continuing with the length scales defined earlier, it is only practical to define a small number of three-dimensional meshes for numerical simulation. The quantities b and c have to be an integral number of multiples of a. The meshes used in our simulations are illustrated in Fig. 3. The top mesh in Fig. 3 requires 196 elements, the two meshes in the second row require 405 elements, and the three in the bottom row require 726 elements. This is a large number of elements: SHAFT79 was originally limited to 501 elements [20].

We examined thermal convection by systematically examining the various components of the Rayleigh number. First we considered the permeability k and the strength of the heat source Q.

5.1. *Permeability and heat input*

Inspection of equations (24)–(26) indicates that $\langle T\rangle$ should be independent of kQ below some critical value, and a function of kQ above this value. This expectation was verified in the numerical experiments. Results with $a = 6$ m, $b = 30$ m and $c = 24$ m are plotted in Fig. 4. It is apparent that there is a critical value of kQ above which convection becomes important. Above the critical Rayleigh number, we observed that τ is approximately proportional to $Q \log kQ$.

5.2. *Gravity*

A set of simulations was performed with all the parameters except gravitational acceleration g held constant. The results of these simulations with various values of g are shown in Fig. 5. In this set of simulations, $a = 6$ m, $b = 30$ m, $c = 24$ m, $k = 3.0 \times 10^{-12}$ m^2 and $Q = 2.0$ kW, so that the Rayleigh number defined by equation (26) is approximately $1.1g$. We observed that convection is insignificant for values of g below 0.1. Above a value of 1.0 convection becomes important, thus the critical value of the Rayleigh number from this data is approximately 1. The curve in Fig. 5 is very similar in shape to Fig. 4, with τ being approximately proportional to $\log g$ above the critical Rayleigh number.

5.3. *Length scales*

As discussed above, there are several length scales that are required to describe a thermosyphon. There is the length scale that describes the outer boundary, b, the length scale for the minor dimension of the loop, a, and the length scale for the side of the loop, c. There is also another length scale. If a thermosyphon is symmetrically placed with respect to the external boundaries, then the heat source is not at the centre, but is offset toward a boundary corner. In presenting the results, we have ignored this offset length scale. With this approximation, the results shown in Figs. 6–9 were obtained from simulations with SHAFT79.

The effect of the distance to the outer boundary, b, was determined while holding a and c constant. When this was done, it appeared that $\tau \propto -1/b$, but this effect was small (Fig. 6). Consequently, the effect of b on convection was ignored, b only being considered for conduction (equation (30)). This considerably simplifies the analysis, because dimensionless groups involving b can be omitted, leaving only a and c.

The effect of uniformly enlarging the thermosyphon

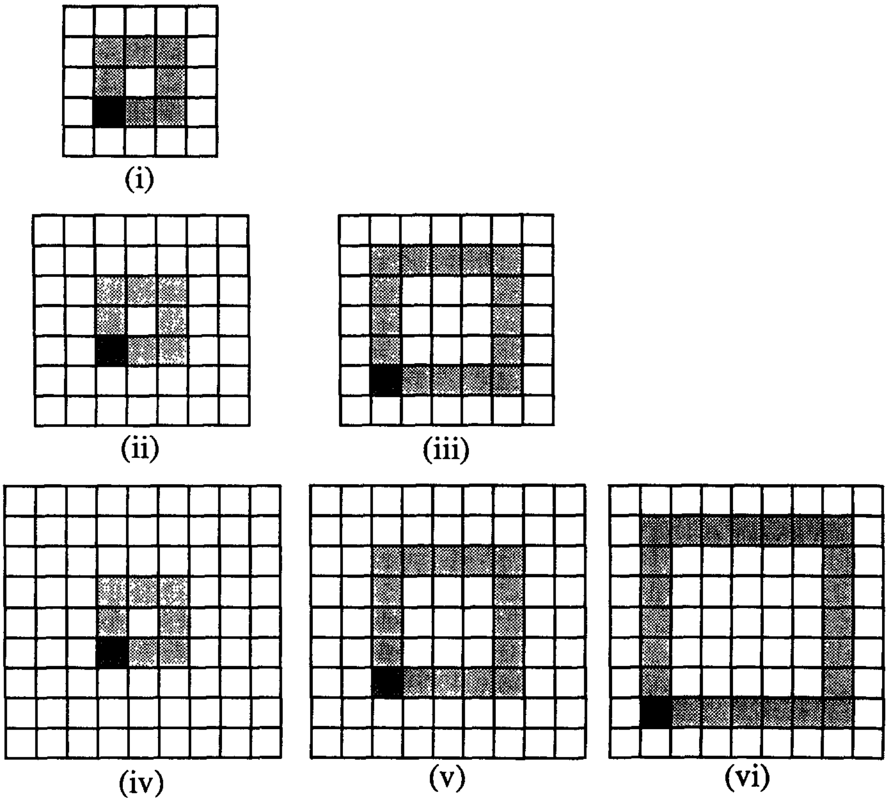

FIG. 3. Cross-sections of the six three-dimensional meshes used in the simulations. The thermosyphon loop is shown in light gray, and the element containing the heat source is shown in dark gray. Mesh (i) requires 196 elements, meshes (ii) and (iii) require 405 elements, and meshes (iv)–(vi) require 726 elements.

is shown in Fig. 7. $a\tau$ is plotted against $\log k$ with $c = 4a$, $Q = 2.0$ kW and $b = 5a$. Changing a and c together so that the ratio a/c remained constant had only a small effect. A pattern is emerging. The results are consistent with an equation of the form

$$\langle T \rangle = \frac{3Q}{8\pi\kappa a}[1 - f(Ra)] \tag{31}$$

where Ra is furnished by equation (26) and the function f is given by

$$f(Ra) = \tfrac{9}{32}\pi^2 \, Ra$$

for small Ra ($Ra < 10$), from equation (27)

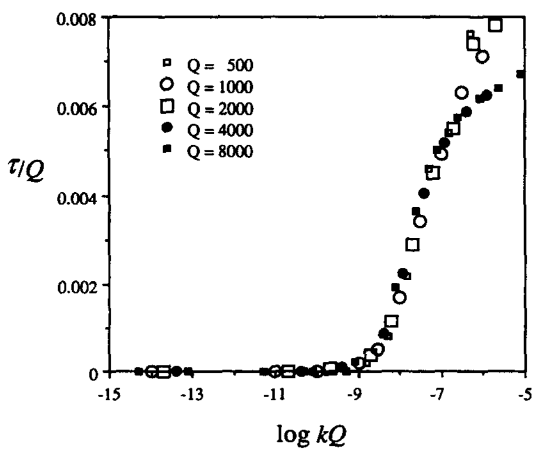

FIG. 4. The effect of permeability and heat input: a plot of τ/Q against $\log kQ$. $a = 6$ m, $b = 30$ m and $c = 24$ m.

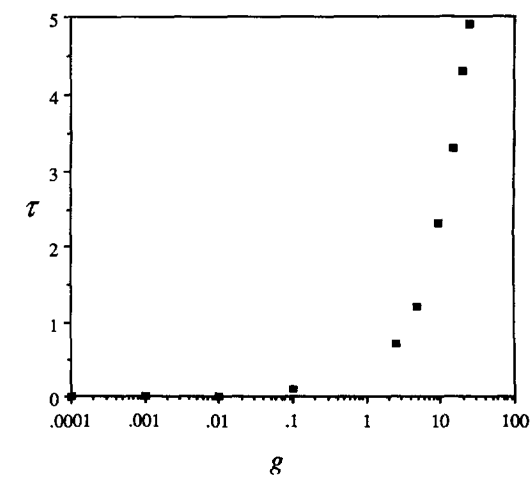

FIG. 5. The effect of gravity: a plot of τ against g. $k = 3 \times 10^{-12}$ m^2, $Q = 2$ kW, $a = 6$ m, $b = 30$ m and $c = 24$ m.

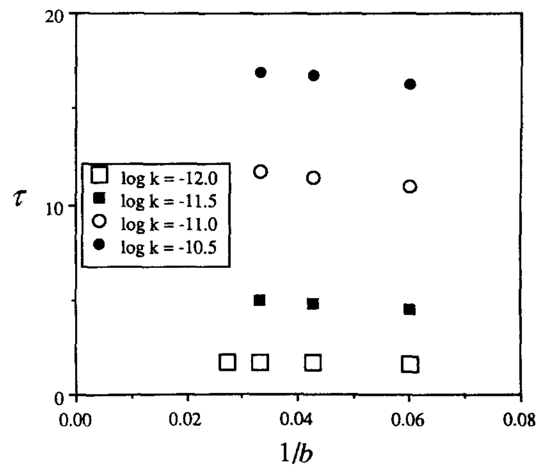

FIG. 6. The effect of the distance to the outer boundary: a plot with a and c fixed, showing the effect of b and k on τ. The effect of b is small. $Q = 2$ kW, $a = 3.33$ m and $c = 13.33$ m.

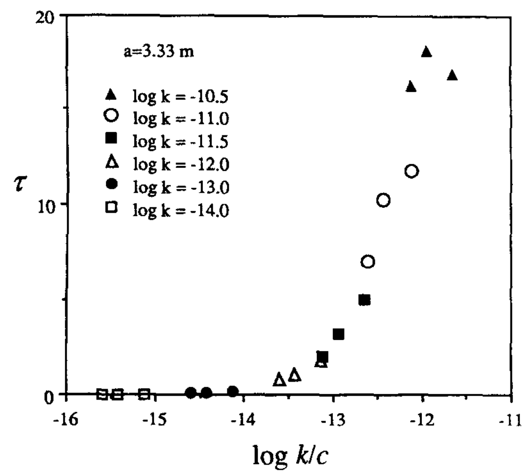

FIG. 8. The effect of the circumference of the thermosyphon: a plot of τ against $\log k/c$ with $a = 3.33$ m, $b = 30.0$ m and $Q = 2.0$ kW.

$$f(Ra) \sim \log Ra$$

for intermediate Ra ($10 < Ra < 1000$).

The effect of the side length of the thermosyphon, c, is shown in Fig. 8, with $a = 3.33$ m, and in Fig. 9, with $a = 10.0$ m. In these figures τ is plotted against $\log k/c$ with $b = 30$ m and $Q = 2.0$ kW. The dependence on the side length, c, is similar to the previous plots. If c is large, then the Rayleigh number is small, and the reduction in $\langle T \rangle$ is insignificant, consistent with the prediction of equation (31). A large loop provides so much resistance that fluid motion is inhibited. As the loop becomes smaller, the reduction in the temperature of the source element begins to become significant. However, as heat transport around the loop improves, the Rayleigh number becomes larger and the approximations described above are no longer valid. In particular, if the heat is able to travel around the loop so rapidly that there is little heat loss before the heat returns to the source element, then the length c has little effect.

The numerical requirement to keep b and c an integral number of a means that the effect of a cannot be determined directly by holding b and c constant without using a prohibitively large number of elements. However, the data displayed in Figs. 8 and 9 can be combined (Fig. 10) to examine the effect of varying a by plotting $a\tau$ against $\log ak/c$. Again the results are consistent with an equation of the form of equation (31).

6. CONCLUSION

We have studied heat flow from an isolated source in a closed geologic thermosyphon using approximate

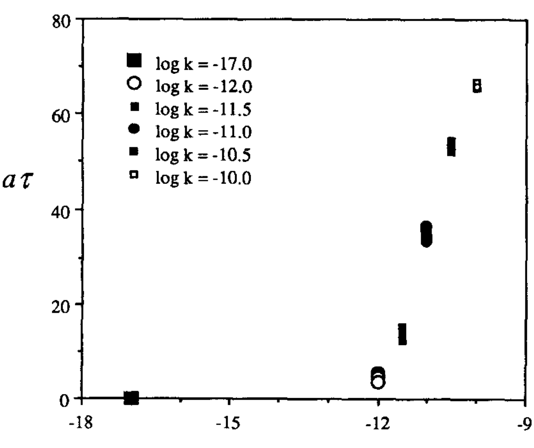

FIG. 7. The effect of uniformly enlarging the thermosyphon: a plot of $a\tau$ against $\log k$ with $c = 4a$, $Q = 2.0$ kW and $b = 5a$. It is apparent that changing a and c together so that the ratio a/c remains constant has little effect.

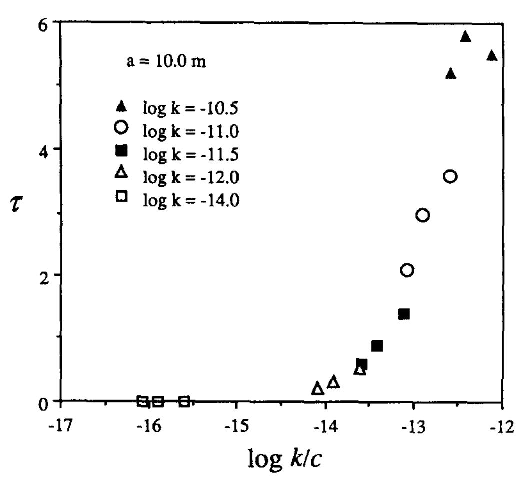

FIG. 9. The effect of the circumference of the thermosyphon; a plot of τ against $\log k/c$ with $a = 10.0$ m, $b = 90.0$ m and $Q = 2.0$ kW.

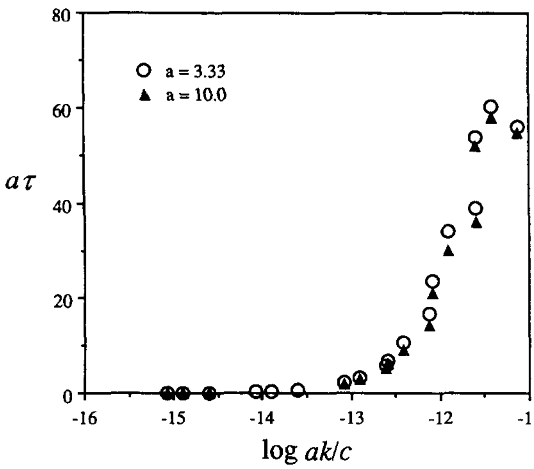

FIG. 10. Combining Figs. 8 and 9: a plot of $a\tau$ against $\log ak/c$ with $Q = 2.0$ kW.

analysis and numerical simulation. A numerical method obtains a lower temperature in the element containing the source if a larger region surrounding the source is averaged. This is a problem associated with using numerical elements that are of the same size as the structural detail. However, this does not influence the determination of the Rayleigh number.

A Rayleigh number definition identical to Bejan's, except for an additional ratio of a/c where a is half the thickness and c the side length of the thermosyphon loop, was effective in describing the performance of a geologic thermosyphon. At a Rayleigh number above 1, convection resulted in a temperature reduction in the vicinity of the source. Temperature reduction could also be obtained by enlarging the cavity containing the source.

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CONVECTION DANS UN THERMOSIPHON POREUX NOYE DANS UN MILIEU CONDUCTEUR

Résumé—Dans un thermosiphon gélogique, la convection thermique dans une boucle fermée est couplée à la conduction dans la terre environnante. On étudie le flux thermique à partir d'une source unique dans ce thermosiphon en utilisant l'analyse approchée et la simulation numérique. On identifie des problèmes particuliers associés avec l'usage des éléments numériques qui sont de la même taille que le détail structurel. Une définition de nombre de Rayleigh identique à celle de Bejan est obtenue, excepté pour un rapport additionnel a/c où a est une dimension caractéristique décrivant la plus petite section droite de la boucle du thermosiphon et c est une dimension caractéristique du plus grand diamètre de la boucle. Pour une valeur du nombre de Rayleigh supérieure à 1, la convection accompagne une réduction de température près de la source.

KONVEKTION IN EINEM PORÖSEN THERMOSYPHON IN EINEM LEITENDEN MEDIUM

Zusammenfassung—In einem geschlossenen geologischen Thermosyphon ist die thermische Konvektion in einem geschlossenen Kreislauf mit der Wärmeleitung im umgebenden Erdreich gekoppelt. Die Wärmeströmung von einer isolierten Quelle in einem solchen porösen Thermosyphon wird unter Verwendung analytischer Näherungen und numerischer Simulationsrechnungen untersucht. Bei der Verwendung numerischer Elemente derselben Größe wie die strukturellen Einzelheiten ergeben sich spezielle Probleme. Es wird eine Definition der Rayleigh-Zahl verwendet, die mit derjenigen von Bejan nahezu übereinstimmt. Der kleine Unterschied besteht in der zusätzlichen Anwendung eines Verhältnisses a/c, wobei a und c charakteristische Abmessungen sind, welche die kleinste bzw. größte Querschnittsfläche der Thermosyphon-Schleife beschreiben. Für Rayleigh-Zahlen oberhalb 1 führt die Konvektion zu einer Temperaturabsenkung in der Nähe der Quelle.

КОНВЕКЦИЯ В ПОРИСТОМ ТЕРМОСИФОНЕ, ПОГРУЖЕННОМ В ПРОВОДЯЩУЮ СРЕДУ

Аннотация—В случае замкнутого геологического термосифона тепловая конвекция в нем связана с проводимостью окружающей почвы. С помощью приближенного анализа и численного моделирования исследуется тепловой поток от изолированного источника в рассматриваемом пористом термосифоне. Решаются конкретные задачи, связанные с использованием метода конечных элементов, размер которых равен размеру структурных деталей. Число Рэлея идентично значению, определяемому Бежаном, за исключением дополнительного отношения a/c, где a и c характерные размеры, соответствующие меньшему и большему сечениям термосифона. При значениях числа Рэлея выше 1 конвекция приводит к понижению температуры вблизи источника.